PROBABILITY AND COUNTING

SEE ALSO: http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut56_perm.htm

Probability is the likelihood that something is going to happen. It is expressed as a percentage from 0% (Impossible) to 100% (Absolutely certain) or as a decimal from 0.0 (Impossible) to 1.0 (Absolutely certain). There are two basic types of probability – theoretical and experimental.

Theoretical Probability is found by the following formula:

$$p(\text{event}) = \frac{\text{\# of ways the event can happen}}{\text{\# of possible outcomes}}$$

Q: A six-sided number cube is rolled. What is the theoretical probability you will roll an odd number?

\# of ways an odd number can be rolled = three {1, 3, 5}
\# of possible outcomes = six {1, 2, 3, 4, 5, 6}

$$p(\text{odd}) = \frac{3}{6} = \frac{1}{2} = 0.50 \text{ or } 50\%$$

Experimental Probability is found by the following formula:

$$p(\text{event}) = \frac{\text{\# of times the event happened}}{\text{\# of attempts}}$$

Q: A six-sided number cube is rolled 20 times. An odd number was rolled 11 of those times. What is the experimental probability that an odd number is rolled?

$$p(\text{odd}) = \frac{11}{20} = 0.55 \text{ or } 55\%$$

THE FUNDAMENTAL COUNTING PRINCIPLE

Q: Golden Corral has an ice cream machine complete with toppings. There are three varieties of ice cream [vanilla, chocolate, swirled], two choices of soft topping [hot fudge, caramel], and three choices of sprinkles topping [rainbow, gummy bears, M&Ms]. How many possible sundaes can be made with one choice of ice cream, hard topping, and soft topping?

UPSL: ~~Golden Corral has an ice cream machine complete with toppings~~//There are three varieties of ice cream [vanilla, chocolate, swirled]//two choices of soft topping [hot fudge, caramel]//and three choices of sprinkles topping [rainbow, gummy bears, M&Ms]//How many possible sundaes can be made with one choice of ice cream, hard topping, and soft topping?

A: Make a tree diagram or use the fundamental counting principle.

Strategy #1: Use The Fundamental Counting Principle

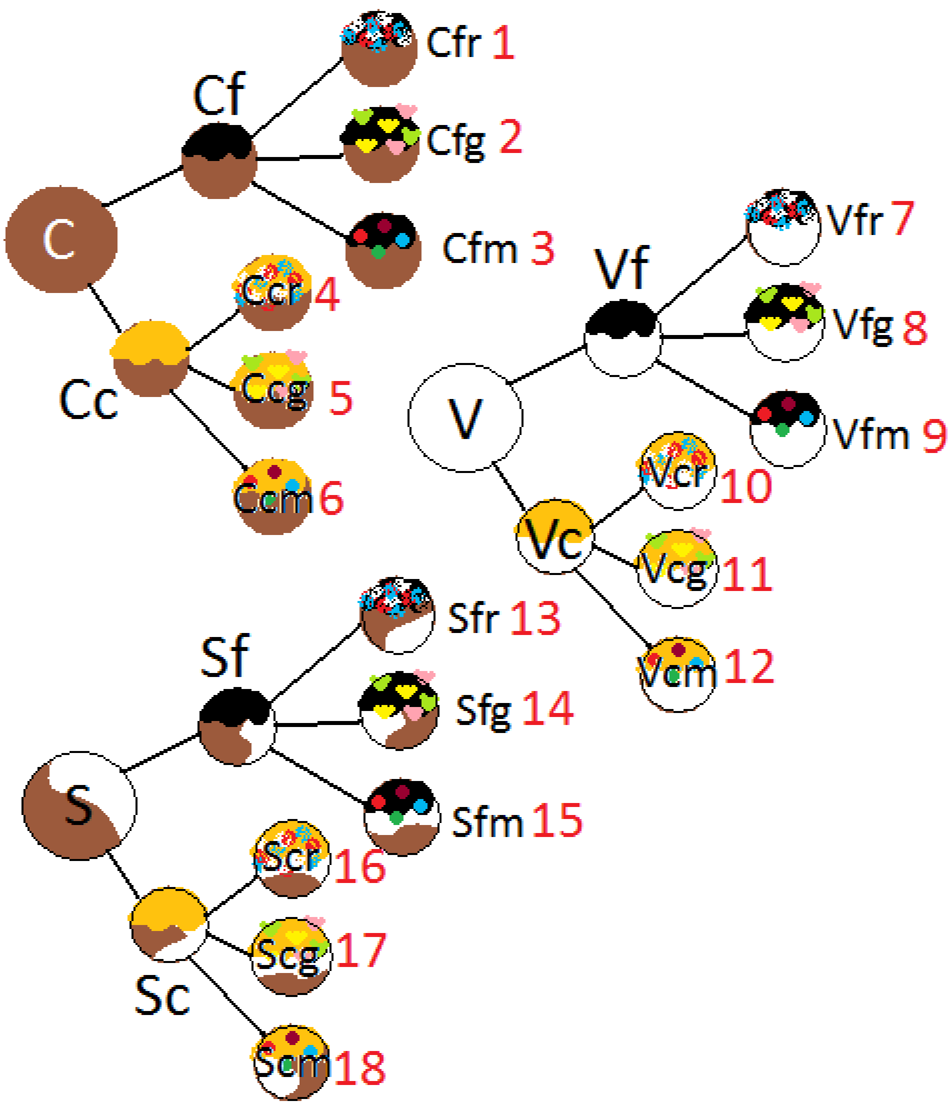
This rule tells us to multiply together the number of items in each section to get the total. If I have three flavors of ice cream, two types of soft topping, and three choices of sprinkles, I simply multiply: 3 • 2 • 3 = 18 sundaes

Stretgy #2: Use A Tree Diagram

Start by giving each option its own letter:

Ice cream: **(C)**hocolate, **(V)**anilla, **(S)**wirled
Soft Topping: hot **(f)**udge, **(c)**aramel
Sprinkles: **(r)**ainbow, **(g)**ummi bears, **(m)**&ms

Starting with our ice cream flavors, we give each one a branch that corresponds to a sprinkle, and each sprinkle gets a branch representing the soft topping.
(See the diagram on the next page)



Counting up the number of branches on the end gives us the number of possible sundaes, which are 18. This is the same result we got using the FCP.

TRY THIS #1 – You have five shirts, three pairs of pants, two hats, and three pairs of shoes in your closet. How many possible outfits can you make?

Answer to TRY THIS #1: 5 x 3 x 2 x 3 = 90 outfits

THE FACTORIAL

Q: On her shelf, Mariah has four pictures. Bored one Saturday afternoon, she rearranged them every possible way she could think of. She found 13 different ways but asks herself: “Are there more ways?”

UPSL: ~~On her shelf, Mariah has four pictures??Bored one Saturday afternoon, she rearranged them every possible way she could think of~~//She found 13 different ways but asks herself: “Are there more ways?”

The Factorial

The factorial is the product of a whole number and every whole number between itself and one and is denoted with an exclamation mark (!). Algebraically, it can be written $n! = n \bullet (n-1) \bullet (n-2) \bullet \ldots \bullet 1$. Here are some numerical examples:

$0! = 1$ (There is one way to choose nothing)

$1! = 1$ $\quad$ $2! = 2 \bullet 1 = 2$ $\quad$ $3! = 3 \bullet 2 \bullet 1 = 6$

$4! = 4 \bullet 3 \bullet 2 \bullet 1 = 24$ $\quad$ $5! = 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1 = 120$

As you can see, the numbers get large pretty quickly. In fact, 8! = 40,320. Factorial is used when arranging a set of items and all items in the set will be used.

A: Since she has four pictures and is using all four in each arrangement, the answer is $4! = 4 \bullet 3 \bullet 2 \bullet 1 = 24$. To answer Mariah’s question, there are more than 13 ways to arrange four pictures.

TRY THIS #2: Six people are going to line up for a picture. In how many different ways can all six of them line up?

Answer to TRY THIS #2: $6! = 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1 = 720$ ways to line up all six people

$_nP_r$ PERMUTATIONS

Q: The school student council has 10 members. They will first choose who will be president, then vice president, secretary, and finally treasurer. In how many ways can five of the ten people be picked for the five offices?

UPSL: The school student council has 10 members//They must choose who will be president, vice president, secretary, and treasurer//In how many ways can five people be picked for the five offices?

A: The order in which these five people are picked matters. When the order the items are arranged in matters, it is a problem involving permutations. The formula for permutations is as follows:

$$_nP_r = \frac{n!}{(n-r)!}$$

Where 'n' is the number of items in the set and 'r' is how many of those items will be picked out from the set. In the case of you problem, there are ten people, so n=10. We are picking out five of them to serve in special offices, so r=5.

Plugging in, we get:

$$_{10}P_5 = \frac{10!}{(10-5)!}$$

$$_{10}P_5 = \frac{10!}{(5)!}$$

$$_{10}P_5 = \frac{10 \bullet 9 \bullet 8 \bullet 7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1}{5 \bullet 4 \bullet 3 \bullet 2 \bullet 1}$$

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SPECIAL NOTE!

In these kinds of problems, you can simplify the problem by renaming any numbers you have on both the top and bottom of the fraction as a '1'.

$$_{10}P_5 = \frac{10 \bullet 9 \bullet 8 \bullet 7 \bullet 6 \cancel{\bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1}}{\cancel{5 \bullet 4 \bullet 3 \bullet 2 \bullet 1}}$$

$$_{10}P_5 = \frac{10 \bullet 9 \bullet 8 \bullet 7 \bullet 6 \bullet 1}{1}$$

$$_{10}P_5 = \frac{30{,}240}{1} = 30{,}240$$

There would be 30,240 different ways that five students could fill the special offices!

TRY THIS #3: Lockers at Milton Middle each have a dial on her locker with 40 numbers. This is mistakenly known as a "combination" lock, though the order the numbers are entered matters. The locker's user must dial three numbers to open their locker. How many locker permutations are possible?

Answer to TRY THIS #3: $_{40}P_3$ = 59,280 possible permutations

nCr COMBINATIONS

Q: Jennifer has six pictures she would like to try in her new frame. The frame can only hold three pictures. She would like to see each possible combination of three pictures to see which one she likes best. How many possible arrangements will she have to try?

UPSL: Jennifer has six pictures she would like to try in her new frame//The frame can only hold three pictures//~~She would like to see each possible combination of three pictures to see which one she likes best~~//How many possible arrangements will she have to try?

A: The order in which the three pictures are chosen really doesn't matter. When the order the items are arranged in doesn't matter, it is a problem involving combinations. The formula for combinations is as follows:

$$_nC_r = \frac{n!}{r!(n-r)!}$$

Where 'n' is the number of items in the set and 'r' is how many of those items will be picked out from the set. In the case of you problem, there are six pictures, so n=6. We are choosing three of them to be put in the frame, so r=3.

Plugging in, we get:

$$_6C_3 = \frac{6!}{3!(6-3)!}$$

$$_6C_3 = \frac{6!}{3!(3)!}$$

$$_6C_3 = \frac{6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1}{3 \bullet 2 \bullet 1 \bullet 3 \bullet 2 \bullet 1}$$

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SPECIAL NOTE!

In these kinds of problems, you can simplify the problem by renaming any numbers you have on both the top and bottom of the fraction as a '1'.

$$_6C_3 = \frac{6 \bullet 5 \bullet 4 \cancel{\bullet 3 \bullet 2 \bullet 1}}{3 \bullet 2 \bullet 1 \cancel{\bullet 3 \bullet 2 \bullet 1}}$$

$$_6C_3 = \frac{6 \bullet 5 \bullet 4}{3 \bullet 2 \bullet 1}$$

$$_6C_3 = \frac{120}{6} = 20$$

There would be 20 different combinations of pictures she could put in the frame.

TRY THIS #4: There are twelve people in the local Women's Club. They need four members to serve on a planning committee. How many possible committees would be possible?

Answer to TRY THIS #3: $_{20}C_4$ = 4,845 possible committees

MEASURES OF CENTRAL TENDENCY

Q: At Central Publishing Co. there are five employees. They make salaries of $100,000, $63,000, $45,000, $45,000, and $35,000. Find the mean, median, mode, and range of the data?

UPSL: At Central Publishing Co. there are five employees. They make salaries of $100,000, $45,000, $63,000, $35,000,and $45,000. Find the mean, median, mode, and range of the data?

A: Different measures of central tendency work better in different situations, but it is always important to use all of them to get a good picture of the data. In any case, you should re-write the data from least-to-greatest:

{35,000, 45,000, 45,000, 65,000, 100,000}

To find the MEAN: Add up the numbers in the set and divide by how many numbers are in the set. There are five numbers in this set:

(100,000 + 65,000 + 45,000 + 45,000 + 35,000) ÷ 5

= (290,000) ÷ 5

= $58,000

The average salary at Central Publishing Co. is $58,000

To find the MEDIAN: Write the data set in order from least to greatest and find the middle number*. *If there is no exact middle number, find the number that is halfway between the two middle numbers.

{35,000, 45,000, <u>45,000</u>, 65,000, 100,000}

The median salary at Central Publishing Co. is $45,000

To find the MODE: Simply find the number (or numbers) that appear most often. If no number shows up more than others, then there is no mode.

{35,000, <u>45,000, 45,000</u>, 65,000, 100,000}

The mode of salaries at Central Publishing Co. is $45,000

To find the RANGE: Simply subtract the smallest value in the set from the largest value in the set.

{<u>35,000</u>, 45,000, 45,000, 65,000, <u>100,000</u>}

The range of salaries at Central Publishing Co. is $100,000 - $35,000 = $65,000